

## MAT-422 SAYILAR TEOR.-II BÜTÜNLEME CEVAP ANAHTARI

$$\boxed{1} \quad 12x \equiv 9 \pmod{75}$$

$(12, 75) = 3 \mid 9$  olduğundan çözüm mod 75-te üç farklı kalan sınıfı çözümlüdür.

$$\begin{aligned} 12x \equiv 9 \pmod{75} &\Rightarrow 4x \equiv 3 \pmod{25} \Rightarrow 24x \equiv 18 \pmod{25} \Rightarrow -x \equiv 18 \pmod{25} \\ &\Rightarrow x \equiv -18 \pmod{25} \\ &\Rightarrow x \equiv \underline{7} \pmod{25} \end{aligned}$$

$$\bar{x}_k = \bar{x}_0 + 25 \cdot k ; k=0, 1, 2 \Rightarrow \bar{x}_1 = \bar{7}, \bar{x}_2 = \bar{32}, \bar{x}_3 = \bar{57}$$

$$\boxed{2} \quad 1+r+r^2+\dots+r^{n-1} = \frac{1-r^n}{1-r} \text{ olduğundan}$$

$$1+2+2^2+\dots+2^n = \frac{1-2^{n+1}}{1-2} = 2^{n+1}-1 \equiv 0 \pmod{77} \Rightarrow 2^{n+1} \equiv 1 \pmod{77}$$

$$(2, 77) = 1 \text{ old. Euler Teo.den: } 2^{\phi(77)} \equiv 2^{60} \equiv 1 \pmod{77} \Rightarrow 66 \mid n+1$$

$$\Rightarrow n+1 = 60 \cdot k, \quad k=2 \Rightarrow \boxed{n=119}$$

$\boxed{3} \quad 56x - 72y = 40$ ,  $(56, 72) = 8 \mid 40$  old. dan verilen Lineer Diophant denkleminin 1-parametreye bağlı sonsuz tamsayı çözümleri mevcuttur.

$$56x - 72y = 40 \Rightarrow 7x - 9y = 5 ;$$

$$9 = 1 \cdot 7 + 2$$

$$7 = 3 \cdot 2 + 1 \Rightarrow 1 = 1 \cdot 7 - 3 \cdot 2$$

$$1 = 1 \cdot 7 - 3 \cdot (9 - 1 \cdot 7)$$

$$1 = 1 \cdot 7 - 3 \cdot 9 + 3 \cdot 7$$

$$\boxed{1 = 4 \cdot 7 - 3 \cdot 9}$$

$$\rightarrow 5 = 7 \cdot (20) - 9 \cdot (15)$$

$$\downarrow$$

$$x_0$$

$$\downarrow$$

$$y_0$$

$$\Rightarrow \left. \begin{aligned} x &= 20 - 9 \cdot t \\ y &= 15 - 7 \cdot t \end{aligned} \right\} \text{ olarak bulunur.}$$

78.153

$$\boxed{14} \text{ a) } \left(\frac{1999}{307}\right) = \left(\frac{157}{307}\right) = (-1) \left(\frac{307}{157}\right) = \left(\frac{307}{157}\right) = \left(\frac{150}{157}\right)$$

$$= \underbrace{\left(\frac{5^2}{157}\right)}_{\textcircled{1}} \cdot \underbrace{\left(\frac{2}{157}\right)}_{157 \equiv 5(8)} \cdot \underbrace{\left(\frac{3}{157}\right)}_{157 \equiv 1(12)}$$

$$\textcircled{-1} \quad \textcircled{1}$$

$$= 1 \cdot (-1) \cdot 1 = -1 //$$

b)

		1	1	2	2	6
0	1	1	2	5	12	77
1	0	1	1	3	7	45

$$\Rightarrow \frac{77}{45} = \langle 1, 1, 2, 2, 6 \rangle$$

II. yOL:

$$1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{6}}}}$$

$$= \frac{77}{45}$$

$$\begin{aligned} 2 + \frac{1}{2 + \frac{1}{6}} &= 2 + \frac{1}{\frac{13}{6}} = 2 + \frac{6}{13} = \frac{32}{13} \\ 1 + \frac{1}{\frac{32}{13}} &= 1 + \frac{13}{32} = \frac{45}{32} \\ 1 + \frac{1}{\frac{45}{32}} &= 1 + \frac{32}{45} = \frac{77}{45} \end{aligned}$$